Comment on "Theory of Spinodal Decomposition"

The basis of Goryachev's analysis [1] of conserved scalar phase-ordering dynamics, to apply only the global constraint $\int \psi \, d\mathbf{x} = \text{const}$, is incorrect. For physical conserved systems, which evolve by mass transport, the stronger local conservation law embodied by the continuity equation

$$\partial\psi/\partial t + \nabla \cdot \mathbf{j} = 0 \tag{1}$$

is the appropriate one to use [2]. Even as an approximation, the global constraint is inadequate [3].

The standard evolution equation for systems with conserved dynamics is

$$\partial\psi/\partial t = \nabla^2 \delta F/\delta\psi, \tag{2}$$

where $F[\psi] = \int d\mathbf{x}[(\nabla\psi)^2 + V_0(\psi^2 - 1)^2]$ is the effective free energy. These dynamics satisfy the local conservation law (1), and are motivated *phenomenologically* by a current $\mathbf{j} = -\nabla\delta F/\delta\psi$. At very early times after a quench from a disordered state, gradients will be large and higher order gradient terms will be needed. Other disagreements with (2) can stem, for example, from hydrodynamic, thermal, and stress relaxation effects. These indicate important extensions needed to (2) and $F[\psi]$; however, the local conservation (1) will still apply in all of these cases.

A special initial condition emphasizes the differences in the microscopic evolution of local versus global conservation, where we only require that the dissipative dynamics be invariant under $\psi \to -\psi$ and that $F[\psi]$ is minimized by $\psi = \pm 1$ everywhere except for a small sphere where $\psi = -1$, the other of which has $\psi = +1$ and -1, respectively. For spheres far from the domain wall, under local conserved dynamics the total magnetization of each half-space will be constant as the spheres evolve. However, with only global conservation always satisfied by the symmetry of the problem, the dynamics are identical to nonconserved dynamics and the magnetization of each half-space will evolve in time and will eventually saturate. This is clearly inconsistent with a local conservation law.

The differences between the global constraint and a local conservation law are also made clear by a class of dynamics introduced by Onuki [4] that includes both cases. In Fourier space we have

$$\partial\psi_{\mathbf{k}}/\partial t = -|\mathbf{k}|^{\sigma} \delta F/\delta\psi_{-\mathbf{k}}, \tag{3}$$

where $\sigma = 2$ is the locally conserved dynamics of (2), $\sigma \to 0^+$ imposes the global constraint discussed by Goryachev, and $\sigma = 0$ is nonconserved dynamics. The differences between local and global conservation laws can be clearly seen in the late-time behavior after a quench, which must be governed by the same nonlinear dynamics as the early-time behavior. As discussed in a unified treatment [5] of (3), and in agreement with previous results [2], the growth laws are $L(t) \sim t^{1/3}$ for (locally) conserved scalar quenches, and $L(t) \sim t^{1/2}$ for nonconserved and globally constrained quenches, where t is the time since the quench. $L(t)$ also describes the radius of the spheres in the previous paragraph, evolving by (3), where t is the time to annihilation.

We can also consider long-range interactions within the effective free energy $F[\psi]$. These are relevant both for attractive [5] and for repulsive [6], or competing, interactions. The free energy should enter into the dynamics the same way, independently of any long-range interactions. This leads to similar differences between local conservation and a global constraint.

Any approximate treatment must start from dynamics that are phenomenologically consistent with microscopic dynamical processes and from effective free energies that are consistent with equilibrium properties. It is incorrect for Goryachev to apply only a global constraint to represent physical systems with local conservation laws.

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